One-Bit Sparse Array DOA Estimation

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Caltech

Outline

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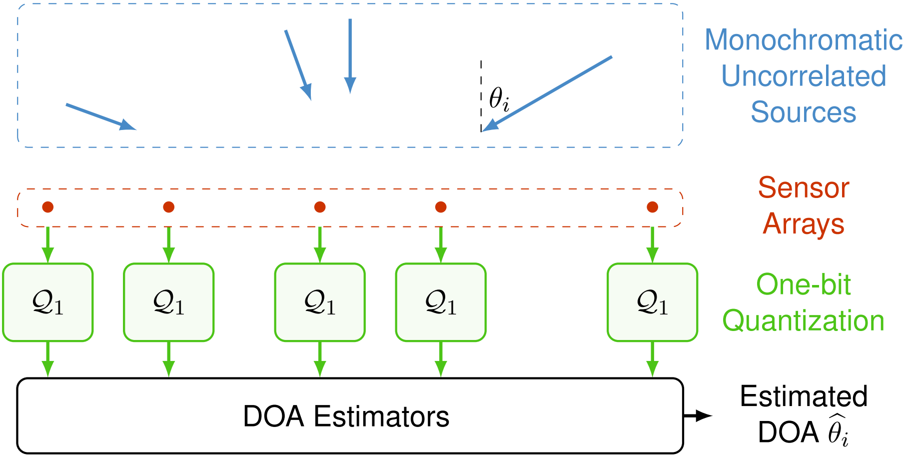
Direction-Of-Arrival (DOA) estimation[1]



[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, 2002.

Sensor arrays and quantization: An example[1]

Array configuration	One-bit quantization	$\text{MSE} = \frac{1}{D}\sum_{i=1}^{D}(\widehat{\bar{\theta}}_i - \bar{\theta}_i)^2$	
Uniform Linear Array 	No	2.525×10^{-6}	(Existing)
	Yes	6.936×10^{-6}	(Proposed)
Nested array 	No	2.077×10^{-7}	(Existing)
	Yes	7.060×10^{-7}	(Proposed)

Nested array + one-bit has smaller MSE than ULA, No Q.

[1] 10 sensors, 5 sources, 0dB SNR, 200 snapshots, equal-power and uncorrelated sources, $\bar{\theta}_i = 0, \pm 0.2, \pm 0.4$, 5000 runs.

ULA and sparse arrays

ULA (not sparse)

- Identify at most $N - 1$ uncorrelated sources, given N sensors.[1]
- Can only find fewer sources than sensors.

Linear sparse arrays

1. Minimum redundancy arrays[2]
2. Nested arrays[3]
3. Coprime arrays[4]
4. Super nested arrays[5]

- Identify $O(N^2)$ uncorrelated sources with $O(N)$ physical sensors.
- More sources than sensors!

[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, 2002.

[2] Moffet, *IEEE Trans. Antennas Propag.*, 1968.

[3] Pal and Vaidyanathan, *IEEE Trans. Signal Proc.*, 2010.

[4] Vaidyanathan and Pal, *IEEE Trans. Signal Proc.*, 2011.

[5] Liu and Vaidyanathan, *IEEE Trans. Signal Proc.*, 2016.

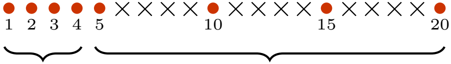
Nested arrays[1]

The nested array

$N_1 = 4,$

$N_2 = 4.$

$|\mathbb{S}| = N_1 + N_2 = 8$



Dense ULA	Sparse ULA
N_1 sensors	N_2 sensors
spacing 1	spacing $N_1 + 1$

Difference coarray

$$\mathbb{D} = \{n_1 - n_2 \mid n_1, n_2 \in \mathbb{S}\} \qquad |\mathbb{D}| = O(N_1 N_2)$$

For sufficient number of snapshots,
$(|\mathbb{U}| - 1)/2 = O(N_1 N_2)$ uncorrelated sources can be identified.
($\mathbb{U}$ = Central ULA part of $\mathbb{D}$)

[1] Pal and Vaidyanathan, *IEEE Trans. Signal Proc.*, 2010.

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One-bit quantization[1]

If x is a real number, then

$$y = \operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ -1, & \text{if } x < 0. \end{cases}$$

If u is a complex number, then

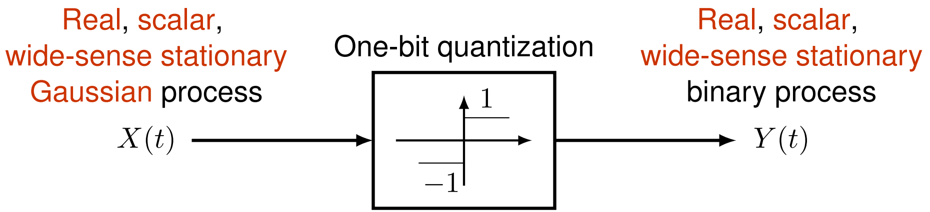
$$\begin{aligned} v &= \mathcal{Q}_1(u) \\ &= \frac{1}{\sqrt{2}}\Big(\operatorname{sgn}(\operatorname{Re}(u)) + j\operatorname{sgn}(\operatorname{Im}(u))\Big). \end{aligned}$$

Advantages

1. Low cost
2. Low complexity
3. Reduced data rate
4. Moderate performance loss
5. Applications in massive MIMO systems

[1] Jacovitti and Neri, *IEEE Trans. Inf. Theory*, 1994; Bar-Shalom and Weiss, *IEEE Trans. Aerosp. Electron. Syst.*, 2002; Lu, Li, Swindlehurst, Ashikhmin, and Zhang, *IEEE J. Sel. Topics in Signal Proc.*, 2014; Larsson, Edfors, Tufvesson, and Marzetta, *IEEE Commun. Mag.*, 2014; Risi, Persson, and Larsson, *arXiv:1404.7736 [cs.IT]*, 2014; Stöckle, Munir, Mezghani, and Nossek, *SPAWC*, 2015; Björnson, Larsson, and Marzetta, *IEEE Commun. Mag.*, 2016; Stein, Barbe, and Nossek, *WSA*, 2016.

Second-order statistics of the quantized data



The arcsine law[1]

$$R_Y(\tau) = \frac{2}{\pi} \sin^{-1}\left(\frac{R_X(\tau)}{R_X(0)}\right),$$

where $R_X(\tau) \triangleq \mathbb{E}[X(t+\tau)X(t)]$ and $R_Y(\tau) \triangleq \mathbb{E}[Y(t+\tau)Y(t)]$.

The Bussgang theorem[2]

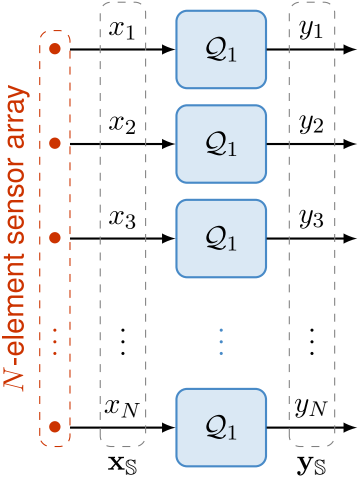
$$R_{XY}(\tau) = C R_X(\tau),$$

where $R_{XY}(\tau) \triangleq \mathbb{E}[X(t+\tau)Y(t)]$ and C is a constant.

[1] Van Vleck and Middleton, *Proc. of the IEEE*, 1966.

[2] Bussgang, *Tech. Rep. 216, Res. Lab. Elec., Mas. Inst. Technol.*, 1952.

The arcsine law for complex vectors



$$\mathbf{R}_{\mathbf{y}_{\mathbb{S}}} = \frac{2}{\pi}\mathrm{sine}^{-1}(\overline{\mathbf{R}}_{\mathbf{x}_{\mathbb{S}}}),$$

where

- $\mathbf{R}_{\mathbf{y}_{\mathbb{S}}}$: the covariance matrix of $\mathbf{y}_{\mathbb{S}}$.
- $\mathrm{sine}^{-1}(\cdot)$: entrywise arcsine function on real and imaginary parts.[2]
- $\overline{\mathbf{R}}_{\mathbf{x}_{\mathbb{S}}} = \mathbf{Q}^{-1/2}\mathbf{R}_{\mathbf{x}_{\mathbb{S}}}\mathbf{Q}^{-1/2}$: Normalized covariance matrix.[3]

[1] Jacovitti and Neri, *IEEE Trans. Inf. Theory*, 1994; Bar-Shalom and Weiss, *IEEE Trans. Aerosp. Electron. Syst.*, 2002.

[2] $[\mathrm{sine}^{-1}(\mathbf{A})]_{p,q} = \sin^{-1}(\mathrm{Re}([\mathbf{A}]_{p,q})) + j\sin^{-1}(\mathrm{Im}([\mathbf{A}]_{p,q}))$.

[3] $\mathbf{Q}$ is a diagonal matrix satisfying $[\mathbf{Q}]_{q,q} = [\mathbf{R}_{\mathbb{S}}]_{q,q}$.

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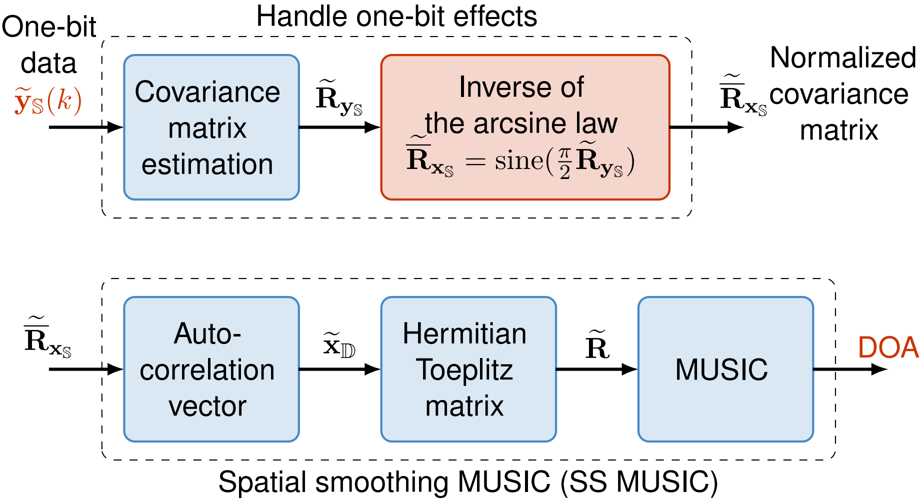
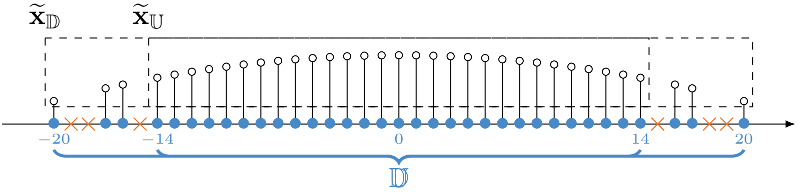
Proposed one-bit DOA estimator for sparse arrays



The spatial smoothing MUSIC Algorithm[1]

1. Sample autocorrelation function on the difference coarray: $\widetilde{\mathbf{x}}_{\mathbb{D}}$.
2. ULA segment of $\widetilde{\mathbf{x}}_{\mathbb{D}}$.



3. Hermitian Toeplitz matrix $\widetilde{\mathbf{R}}$ (indefinite matrix).

$$\widetilde{\mathbf{R}} = \begin{bmatrix} \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_0 & \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_{-1} & \cdots & \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_{-14} \\ \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_1 & \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_0 & \cdots & \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_{-13} \\ \vdots & \vdots & \ddots & \vdots \\ \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_{14} & \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_{13} & \cdots & \langle\widetilde{\mathbf{x}}_{\mathbb{U}}\rangle_0 \end{bmatrix}$$

4. MUSIC on $\widetilde{\mathbf{R}}$ resolves $(|\mathbb{U}|-1)/2 = O(N^2)$ uncorrelated sources.

[1] Pal and Vaidyanathan, *IEEE Trans. Signal Proc.*, 2010; Liu and Vaidyanathan, *IEEE Signal Proc. Letters*, 2015.

Normalized covariance matrix in SS MUSIC

- SS MUSIC: the original covariance matrix $\mathbf{R}_{\mathbf{x}_{\mathbb{S}}}$.
- One-bit data: the normalized covariance matrix $\overline{\mathbf{R}}_{\mathbf{x}_{\mathbb{S}}}$.

Lemma

If the source amplitudes are uncorrelated, then

$$\mathbf{R}_{\mathbf{x}_{\mathbb{S}}} = P\overline{\mathbf{R}}_{\mathbf{x}_{\mathbb{S}}},$$

where $P = \sum_{i=1}^{D} p_i + p_n$ *is the total power.*

For Gaussian uncorrelated sources and sufficient snapshots, we have

1. Eigenvalues of $\mathbf{R}_{\mathbf{x}_{\mathbb{S}}} = P \times$ Eigenvalues of $\overline{\mathbf{R}}_{\mathbf{x}_{\mathbb{S}}}$
2. Eigenspace of $\mathbf{R}_{\mathbf{x}_{\mathbb{S}}} =$ Eigenspace of $\overline{\mathbf{R}}_{\mathbf{x}_{\mathbb{S}}}$
3. They share the same SS MUSIC spectra.

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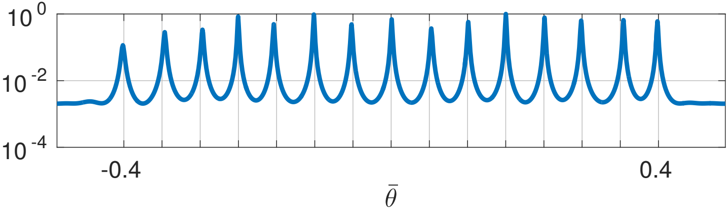
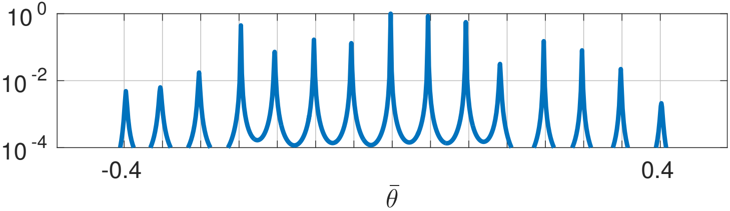
One-bit MUSIC spectra (10 sensors, 15 sources)[1]

Nested array $(N_1 = N_2 = 5)$; $\mathrm{MSE} = 6.2203 \times 10^{-6}$.

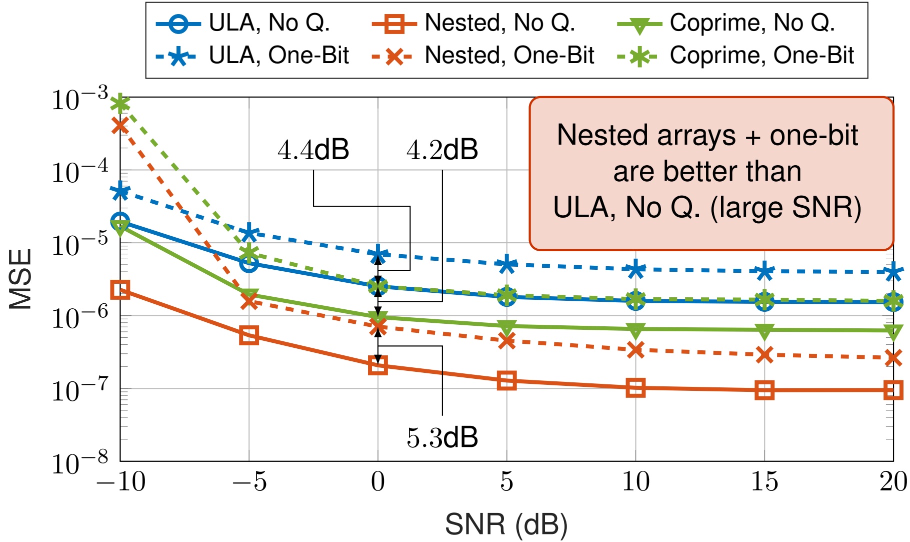




Coprime array $(M = 3, N = 5)$; $\mathrm{MSE} = 1.5816 \times 10^{-5}$.

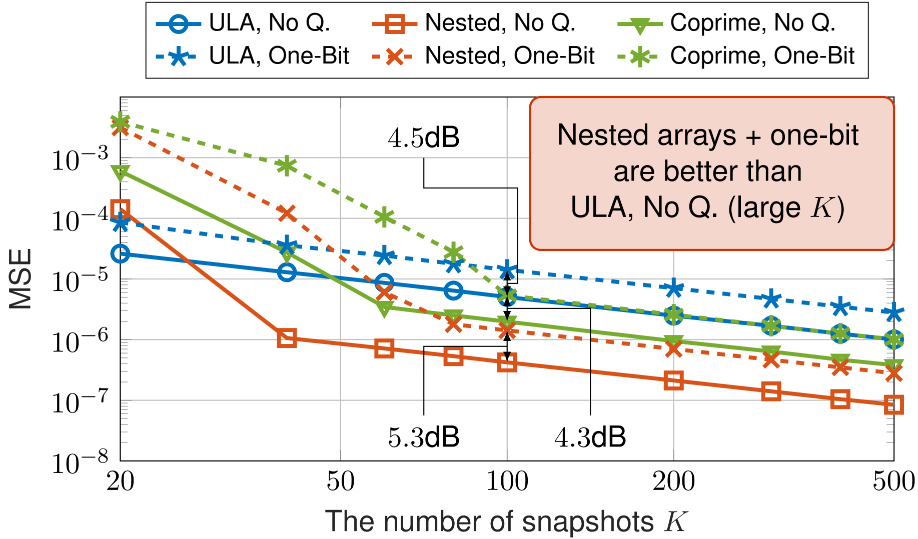
[1] 200 snapshots, 0dB SNR, equal-power and uncorrelated sources, $\mathrm{MSE} = \sum_{i=1}^{D} (\widehat{\bar{\theta}}_i - \bar{\theta}_i)^2 / D$.

MSE vs SNR (10 sensors, 5 sources)[1]



[1] 200 snapshots, equal-power and uncorrelated sources, $\bar{\theta}_i = 0, \pm 0.2, \pm 0.4$, $\mathrm{MSE} = \sum_{i=1}^{D} (\widehat{\bar{\theta}}_i - \bar{\theta}_i)^2 / D$.

MSE vs snapshots (10 sensors, 5 sources)[1]



[1] 0dB SNR, equal-power and uncorrelated sources, $\bar{\theta}_i = 0, \pm 0.2, \pm 0.4$, $\text{MSE} = \sum_{i=1}^{D} (\widehat{\bar{\theta}}_i - \bar{\theta}_i)^2 / D$.

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Concluding remarks

- DOA estimator with
  - uncorrelated complex Gaussian sources,
  - sparse arrays, and
  - one-bit quantized data.
- Empirically, sparse arrays with one-bit quantization can perform better than ULA without quantization for large SNR or sufficient snapshots.
- Future work: Performance analysis.
- Acknowledgement: the plenary talk on massive MIMO systems by Prof. Swindlehurst at the IEEE SAM workshop in 2016.
- Work supported by Office of Naval Research.

Thank you!