Two-Dimensional Sparse Arrays with Hole-Free Coarray and Reduced Mutual Coupling

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Outline

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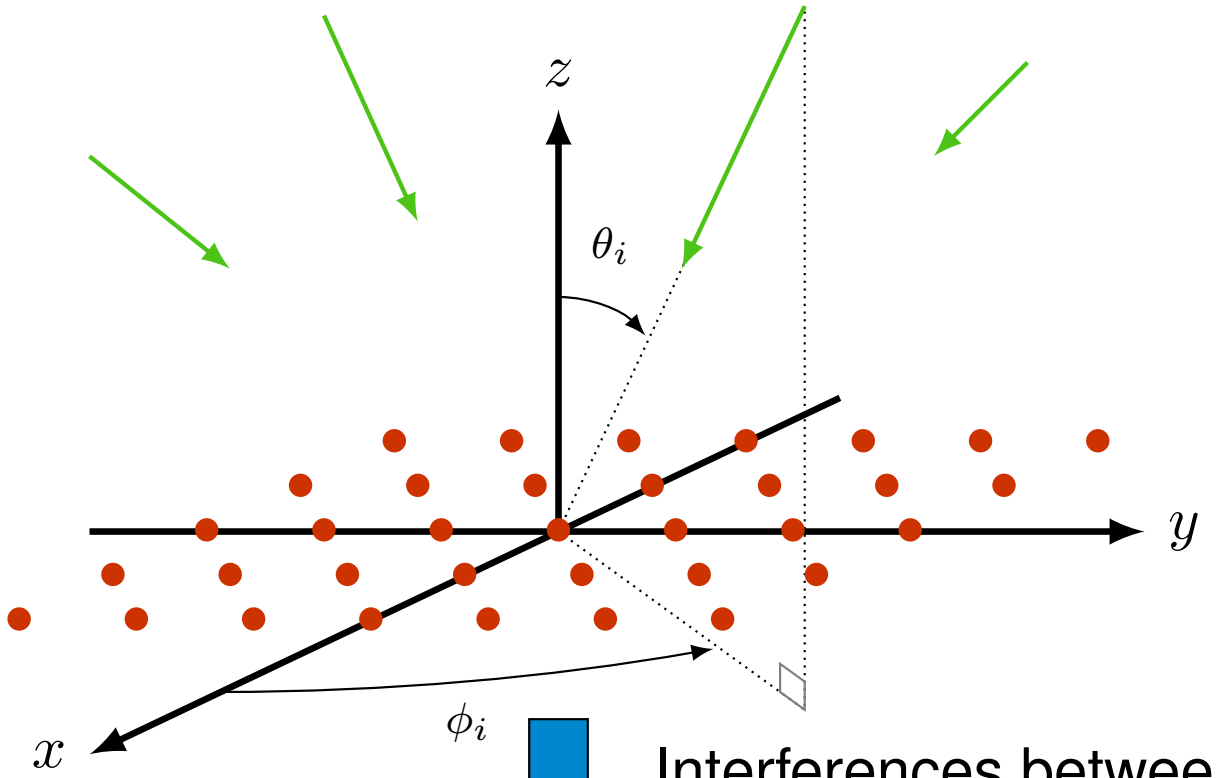
DOA estimation in the presence of mutual coupling[1]



Monochromatic, Far-Field, Uncorrelated Sources

Sensor Array

Interferences between closely-spaced sensors

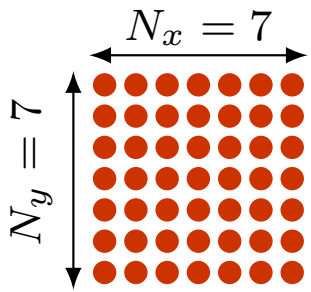
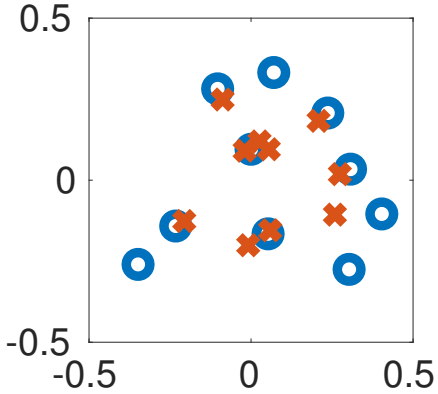
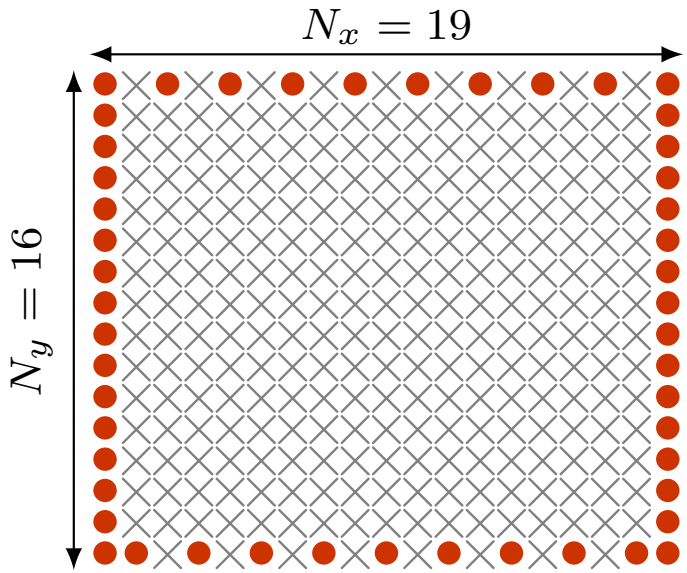
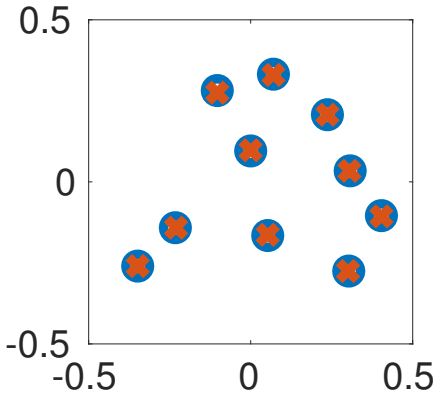
Mutual Coupling

DOA Estimators

Estimated DOA

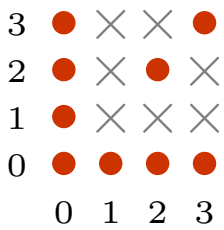
[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, 2002.

URA and the proposed array: An example

	Array Configuration	DOA estimation in the presence of mutual coupling
Uniform Rectangular Array (URA) [49 sensors]		
Half Open Box Array (HOBA) [49 sensors] ***Proposed***		 o: True ×: Estimated 10 sources

Array design criteria: Hole-free difference coarray

Physical Array $\mathbb{S}$



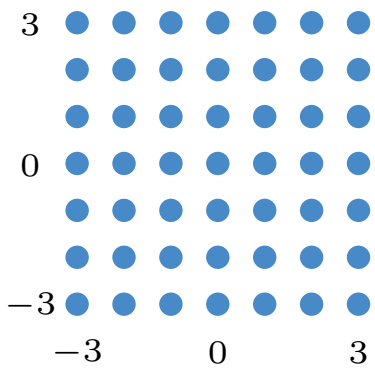
9 elements

$O(N)$

Differences between sensor locations

$$\mathbb{D} = \{\mathbf{n}_1 - \mathbf{n}_2 \mid \mathbf{n}_1, \mathbf{n}_2 \in \mathbb{S}\}$$

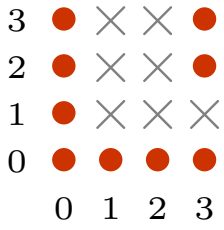
Difference Coarray $\mathbb{D}$



49 elements $O(N^2)$

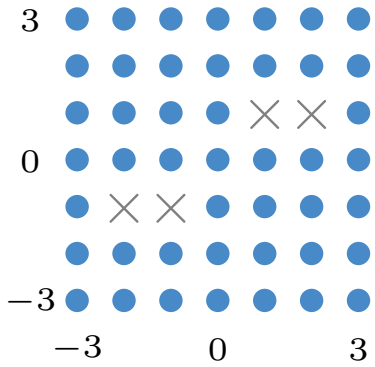
$\mathbb{D}$ is hole-free ($\mathbb{D}$ = URA) ✔

MUSIC or ESPRIT on $\mathbb{D}$



9 elements

× Hole



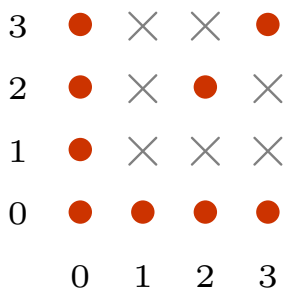
45 elements

$\mathbb{D}$ has holes ✘

Fewer elements in $\mathbb{D}$

Array design criteria: Weight functions[1]

$w(m_x, m_y)$: The number of sensor pairs with separation (m_x, m_y).

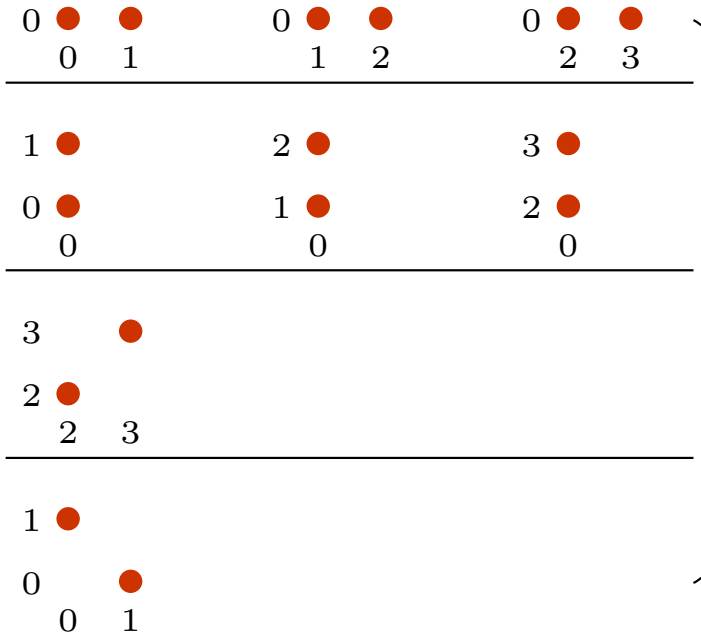


$w(1,0) = 3$

$w(0,1) = 3$

$w(1,1) = 1$

$w(1,-1) = 1$



These pairs cause significant mutual coupling

Small $w(1,0), w(0,1), w(1,\pm 1)$ lead to reduced mutual coupling

[1] Liu and Vaidyanathan, *IEEE Trans. Signal Proc.*, 2016

Outline

Known planar arrays

- Uniform Rectangular Arrays (URA)[1]
- Billboard Arrays[2]
- 2D Nested Arrays[3]
- Open Box Arrays (OBA)[4]

How to rate these arrays?

Fixed number of sensors

- The difference coarray should be hole-free. ☆
- $w(1,0)$ should be small. ☆
- $w(0,1)$ should be small. ☆
- $w(1,1)$ should be small. ☆
- $w(1,-1)$ should be small. ☆

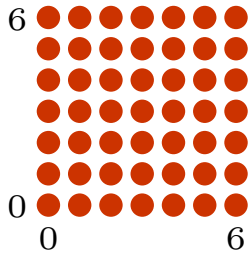
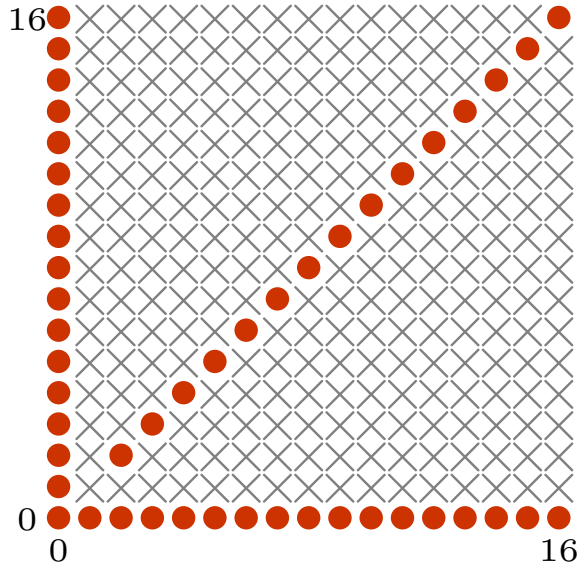
[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, 2002.

[2] Greene and Wood, *J. Acoust. Soc. Am.*, 1978.

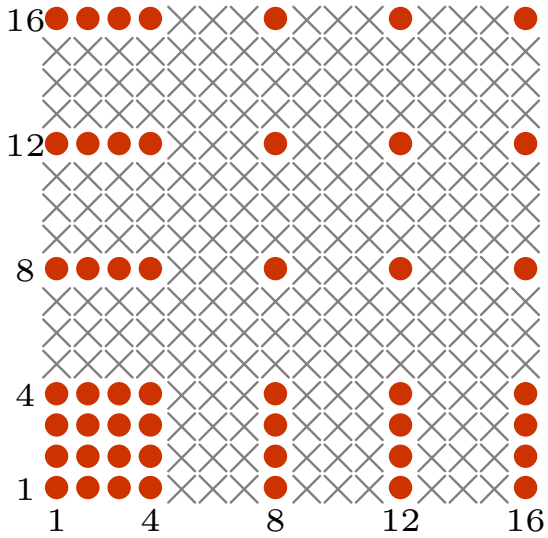
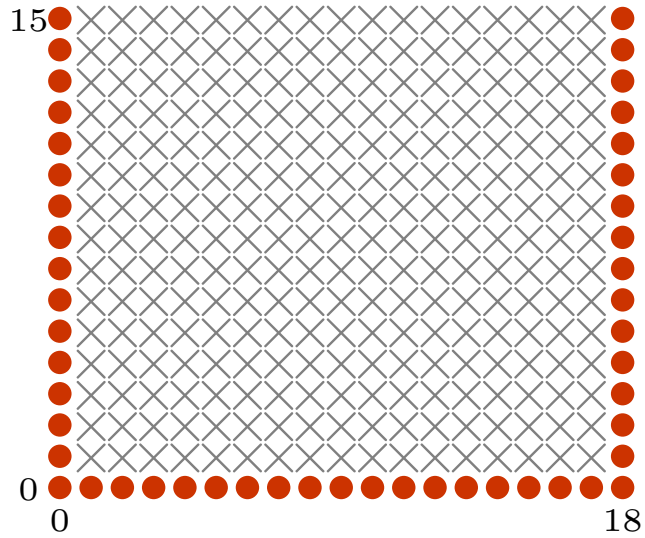
[3] Pal and Vaidyanathan, *IEEE Trans. Signal Process.*, 2012.

[4] Haubrich, *Seismol. Soc. Am., Bull.*, 1968.

Known arrays: URA and billboard array

	URA (49 sensors)	Billboard array (48 sensors)
Array geometry		
$\mathbb{D}$	Hole-free ★	Hole-free ★
$w(1,0)$	42 ✗	16 ✗
$w(0,1)$	42 ✗	16 ✗
$w(1,1)$	36 ✗	14 ✗
$w(1,-1)$	36 ✗	1 ★
Overall rating	★	★★

Known arrays: 2D nested array and OBA

	2D nested array (49 sensors)	OBA (49 sensors)
Array geometry		
$\mathbb{D}$	Hole-free ★	Hole-free ★
$w(1,0)$	21 ✗	18 ✗
$w(0,1)$	21 ✗	30 ✗
$w(1,1)$	9 ★	1 ★
$w(1,-1)$	9 ★	1 ★
Overall rating	★★★	★★★

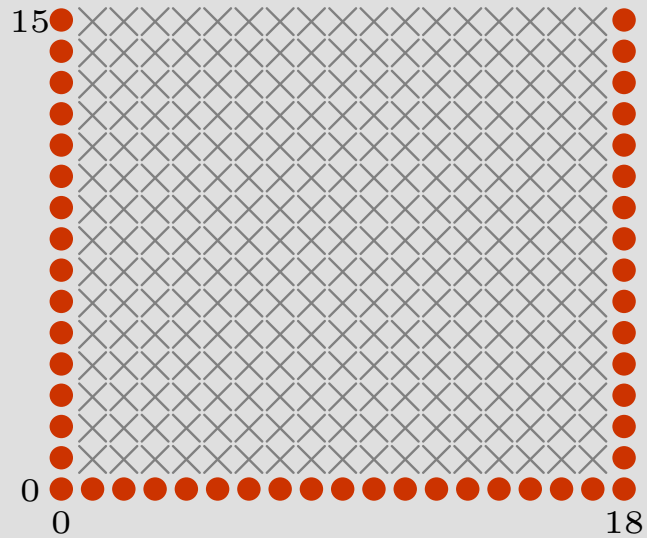
Outline

Main Question: Are there any five-star arrays?

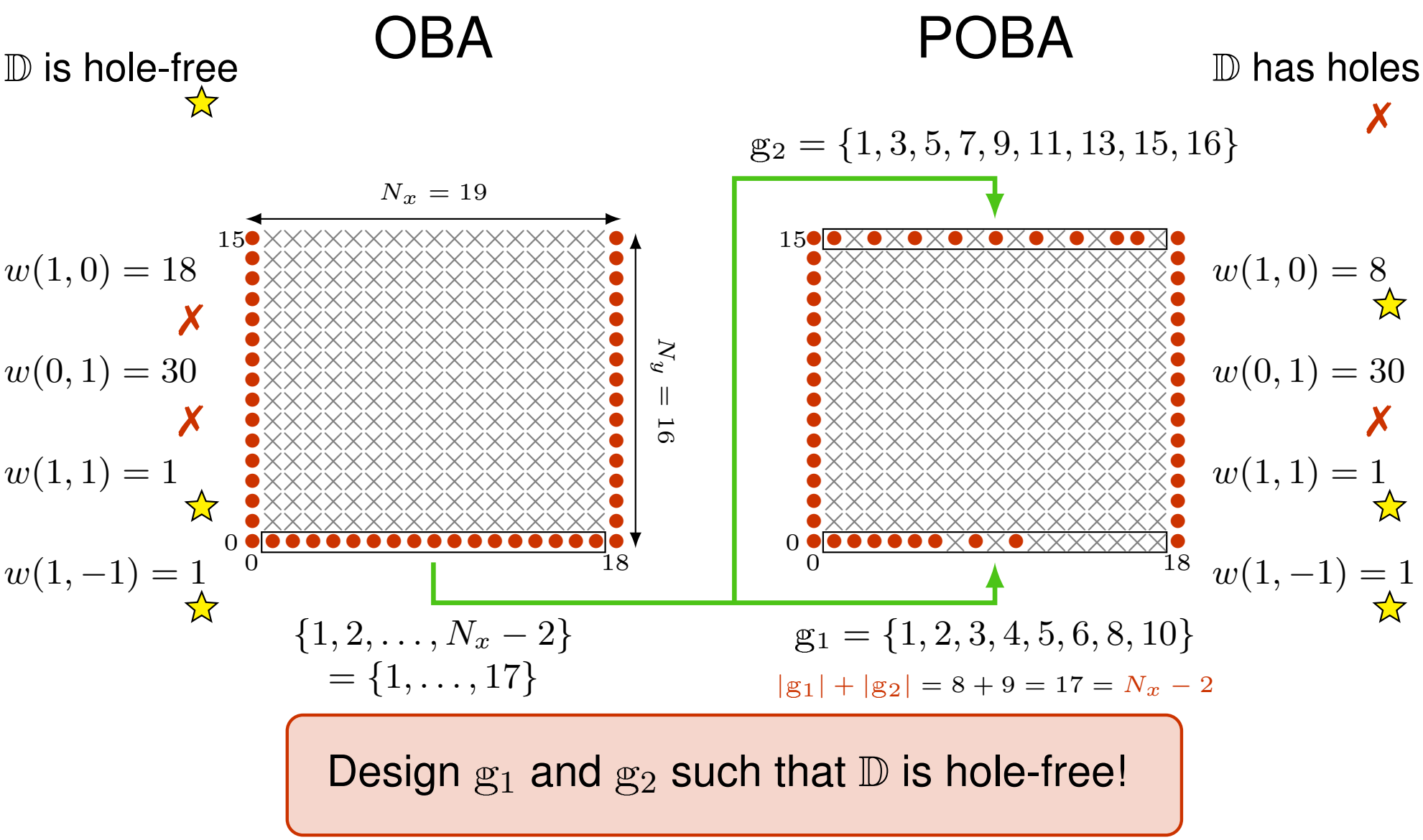
Main idea

Start with a good prototype array and redistribute the sensors.

OBA ★★★



Partially Open Box Arrays: An example

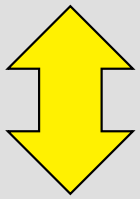


Design g_1 and g_2 such that $\mathbb{D}$ is hole-free!

The exact condition for POBA with hole-free coarrays

Theorem

$$\mathbb{D}_{\text{POBA}} = \mathbb{D}_{\text{OBA}}$$
(In particular, no holes)



if and only if

$$\{\mathbb{g}_1, N_x - 1 - \mathbb{g}_2\}$$
is a partition of
$$\{1, 2, \ldots, N_x - 2\}.$$

- $N_x - 1 - \mathbb{g}_2 \triangleq \{N_x - 1 - g \mid g \in \mathbb{g}_2\}$.
- $\{\mathbb{A}, \mathbb{B}\}$ is a partition of $\mathbb{C}$ if and only if $\mathbb{A} \cap \mathbb{B} =$ the empty set and $\mathbb{A} \cup \mathbb{B} = \mathbb{C}$.

Implications

- Previous example:

  $\mathbb{g}_1 = \{1, 2, 3, 4, 5, 6, 8, 10\}$,

  $N_x - 1 - \mathbb{g}_2 = \{2, 3, 5, 7, 9, 11, 13, 15, 17\}$.
- The hole-free property can be readily checked without computing the coarray.
- POBA with hole-free coarrays:

  Step 1 :

  Choose $\mathbb{g}_1 \subseteq \{1, \ldots, N_x - 2\}$.

  Step 2 :

  $\mathbb{g}_2 = \{1, \ldots, N_x - 2\}$
  $\backslash (N_x - 1 - \mathbb{g}_1)$.

Half Open Box Arrays (Proposed)

Main idea

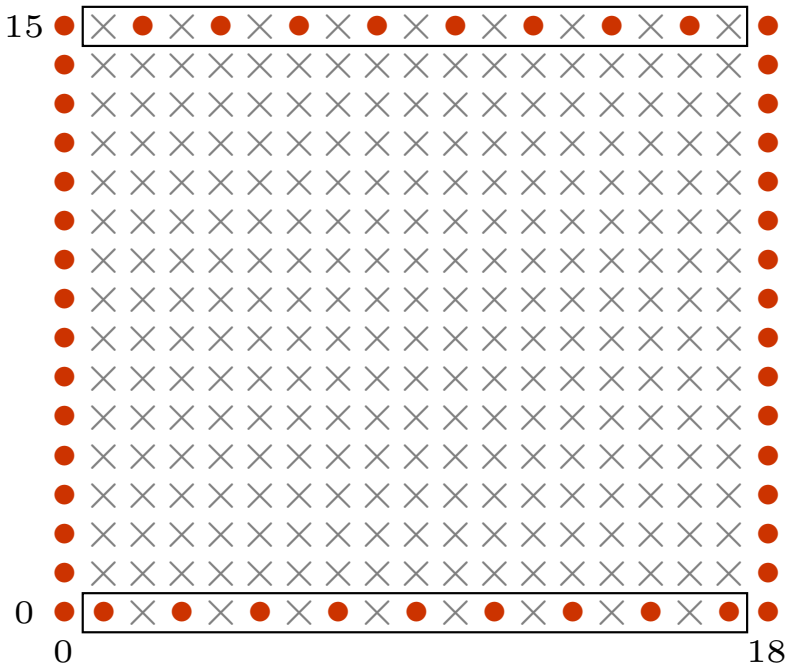
1. Select $\mathbb{g}_1$ to be a ULA with separation 2.

$$\mathbb{g}_1 = \{1 + 2\ell \mid 0 \leq \ell \leq \lfloor (N_x - 3)/2 \rfloor\}.$$

2. Choose $\mathbb{g}_2$ such that the hole-free property holds.

$$\mathbb{g}_2 = \{N_x - 1 - 2\ell \mid 1 \leq \ell \leq \lfloor (N_x - 2)/2 \rfloor\}.$$

$\mathbb{g}_2 = \{2, 4, 6, 8, 10, 12, 14, 16\}$



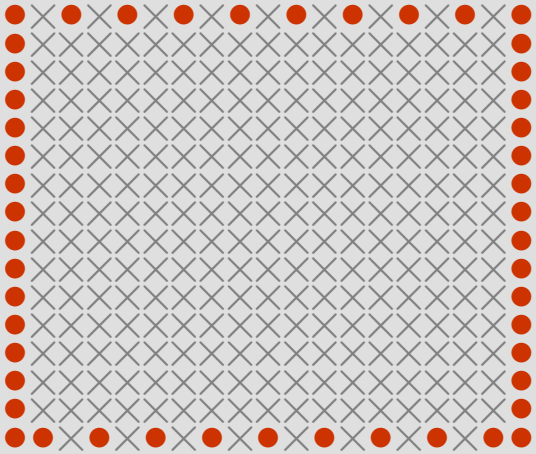
$\mathbb{g}_1 = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$

OBA: $w(1, 0) = 18,$

HOBA: $w(1, 0) = 2.$

Theoretical Guarantees

HOBA ★★★★



- Hole-free $\mathbb{D}$ ★
- $w(1,0) = 2$ ★
- $w(0,1) = 2(N_y - 1) = 30$ ✗
- $w(1,1) = 1$ ★
- $w(1,-1) = 1$ ★

Comparison

- URA ★
- Billboard arrays ★★
- 2D nested arrays ★★★
- OBA ★★★
- HOBA (Proposed) ★★★★

Outline

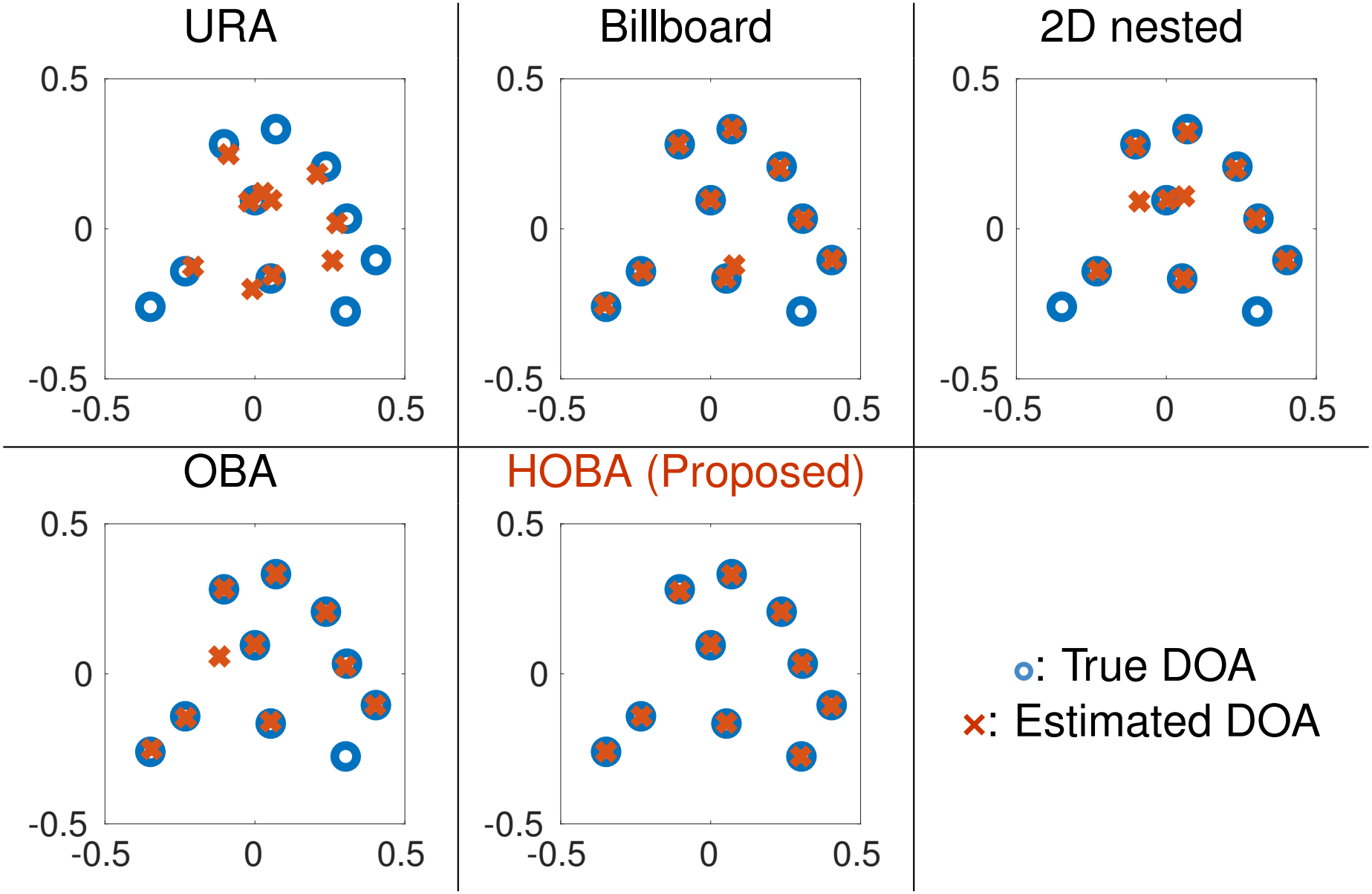
Simulation parameters

- 10 sources, 49 sensors. (The billboard array has only 48 sensors)
- Equal-power, SNR is 0dB.
- 100 snapshots.
- DOA estimation in the presence of mutual coupling.
- 2D unitary ESPRIT on the difference coarray.[1]

> Mutual coupling is present in the measurements,
> but it is not compensated by 2D unitary ESPRIT!

[1] Zoltowski, Haardt, and Mathews, *IEEE Trans. Signal Process.*, 1996.

2D DOA estimation in the presence of mutual coupling



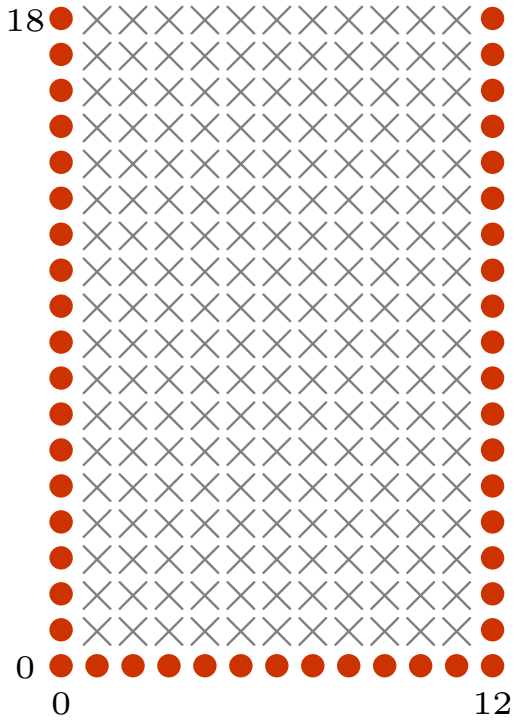
Outline

Concluding remarks

- Half open box arrays (HOBA) ★★★★
  - They have the same number of sensors as open box arrays (OBA).
  - They have the same difference coarray as OBA.
  - They have reduced mutual coupling than OBA.
- For more information, please visit our project website:
  `http://systems.caltech.edu/dsp/students/clliu/`
- In the future, decoupling algorithms will improve the performance.[1]
- Are there any five-star arrays? ★★★★★

[1] Friedlander and Weiss, *IEEE Trans. Antennas Propag.*, 1991; BouDaher, Ahmad, Amin, and Hoorfar, *EUSIPCO*, 2015.

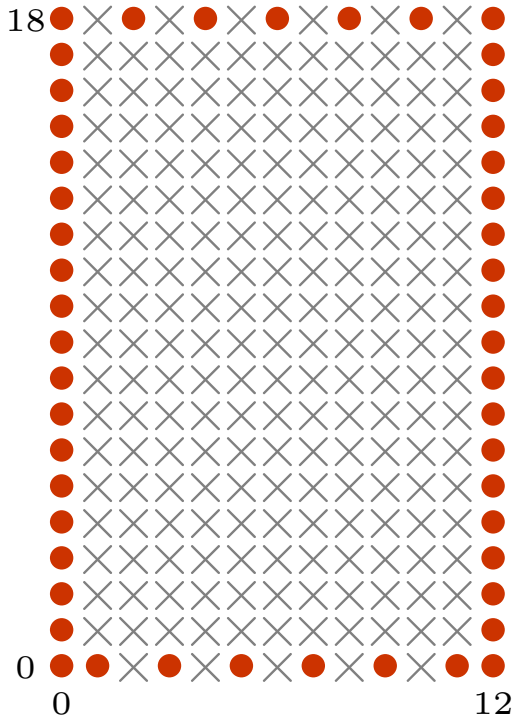
Ongoing work: Hourglass arrays[1]



OBA

49 sensors

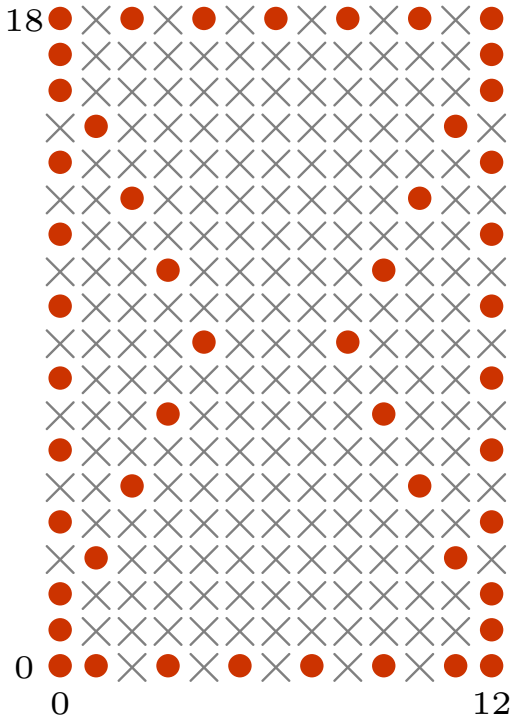
Severe mutual coupling



HOBA

49 sensors

Less mutual coupling



Hourglass Array

49 sensors

Least mutual coupling

[1] Liu and Vaidyanathan, *IEEE Trans. Signal Process.*, 2016, under review.

Thank you!